

KEY CONCEPTS

1. DEFINITION :

If f & g are functions of x such that $g'(x) = f(x)$ then the function g is called a **PRIMITIVE OR ANTIDERIVATIVE OR INTEGRAL** of $f(x)$ w.r.t. x and is written symbolically as

$$\int f(x) dx = g(x) + c \Leftrightarrow \frac{d}{dx} \{g(x) + c\} = f(x), \text{ where } c \text{ is called the constant of integration.}$$

2. STANDARD RESULTS :

$$(i) \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c \quad n \neq -1$$

$$(ii) \int \frac{dx}{ax+b} = \frac{1}{a} \ln(ax+b) + c$$

$$(iii) \int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

$$(iv) \int a^{px+q} dx = \frac{1}{p} \frac{a^{px+q}}{\ln a} \quad (a > 0) + c$$

$$(v) \int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + c$$

$$(vi) \int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + c$$

$$(vii) \int \tan(ax+b) dx = \frac{1}{a} \ln \sec(ax+b) + c$$

$$(viii) \int \cot(ax+b) dx = \frac{1}{a} \ln \sin(ax+b) + c$$

$$(ix) \int \sec^2(ax+b) dx = \frac{1}{a} \tan(ax+b) + c$$

$$(x) \int \operatorname{cosec}^2(ax+b) dx = -\frac{1}{a} \cot(ax+b) + c$$

$$(xi) \int \sec(ax+b) \cdot \tan(ax+b) dx = \frac{1}{a} \sec(ax+b) + c$$

$$(xii) \int \operatorname{cosec}(ax+b) \cdot \cot(ax+b) dx = -\frac{1}{a} \operatorname{cosec}(ax+b) + c$$

$$(xiii) \int \sec x dx = \ln(\sec x + \tan x) + c \quad \text{OR} \quad \ln \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) + c$$

$$(xiv) \int \operatorname{cosec} x dx = \ln(\operatorname{cosec} x - \cot x) + c \quad \text{OR} \quad \ln \tan \frac{x}{2} + c \quad \text{OR} \quad -\ln(\operatorname{cosec} x + \cot x)$$

$$(xv) \int \sinh x dx = \cosh x + c \quad (xvi) \int \cosh x dx = \sinh x + c \quad (xvii) \int \operatorname{sech}^2 x dx = \tanh x + c$$

$$(xviii) \int \operatorname{cosech}^2 x dx = -\coth x + c$$

$$(xix) \int \operatorname{sech} x \cdot \tanh x dx = -\operatorname{sech} x + c$$

$$(xx) \int \operatorname{cosech} x \cdot \coth x dx = -\operatorname{cosech} x + c$$

$$(xxi) \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$$

$$(xxii) \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$(xxiii) \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + c$$

$$(xxiv) \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left[x + \sqrt{x^2 + a^2} \right] \quad \text{OR} \quad \sinh^{-1} \frac{x}{a} + c$$

$$(xxv) \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left[x + \sqrt{x^2 - a^2} \right] \quad \text{OR} \quad \cosh^{-1} \frac{x}{a} + c$$

$$(xxvi) \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \frac{a+x}{a-x} + c$$

$$(xxvii) \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \frac{x-a}{x+a} + c$$

$$(xxviii) \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$$

$$(xxix) \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a} + c$$

$$(xxx) \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a} + c$$

$$(xxxi) \int e^{ax} \cdot \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c$$

$$(xxxii) \int e^{ax} \cdot \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c$$

3. TECHNIQUES OF INTEGRATION :

(i) Substitution or change of independent variable .

Integral $I = \int f(x) dx$ is changed to $\int f(\phi(t)) f'(t) dt$, by a suitable substitution $x = \phi(t)$ provided the later integral is easier to integrate .

(ii) Integration by part : $\int u \cdot v dx = u \int v dx - \int \left[\frac{du}{dx} \int v dx \right] dx$ where u & v are differentiable

function . **Note :** While using integration by parts, choose u & v such that

$$(a) \int v dx \text{ is simple } \& \quad (b) \int \left[\frac{du}{dx} \int v dx \right] dx \text{ is simple to integrate.}$$

This is generally obtained, by keeping the order of u & v as per the order of the letters in **ILATE**, where ; I – Inverse function, L – Logarithmic function,

A – Algebraic function, T – Trigonometric function & E – Exponential function

(iii) Partial fraction , splitting a bigger fraction into smaller fraction by known methods .

4. INTEGRALS OF THE TYPE :

$$(i) \int [f(x)]^n f'(x) dx \quad \text{OR} \quad \int \frac{f'(x)}{[f(x)]^n} dx \quad \text{put } f(x) = t \text{ \& proceed .}$$

$$(ii) \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}, \int \sqrt{ax^2 + bx + c} dx$$

Express $ax^2 + bx + c$ in the form of perfect square & then apply the standard results .

$$(iii) \int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx .$$

Express $px + q = A$ (differential co-efficient of denominator) + B .

$$(iv) \int e^x [f(x) + f'(x)] dx = e^x \cdot f(x) + c \quad (v) \int [f(x) + xf'(x)] dx = x f(x) + c$$

$$(vi) \int \frac{dx}{x(x^n + 1)} \quad n \in \mathbb{N} \quad \text{Take } x^n \text{ common \& put } 1 + x^{-n} = t .$$

$$(vii) \int \frac{dx}{x^2(x^n + 1)^{(n-1)/n}} \quad n \in \mathbb{N}, \text{ take } x^n \text{ common \& put } 1 + x^{-n} = t^n$$

(viii) $\int \frac{dx}{x^n (1+x^n)^{1/n}}$ take x^n common as x and put $1+x^{-n}=t$.

(ix) $\int \frac{dx}{a+b \sin^2 x}$ OR $\int \frac{dx}{a+b \cos^2 x}$ OR $\int \frac{dx}{a \sin^2 x + b \sin x \cos x + c \cos^2 x}$

Multiply N^r & D^r by $\sec^2 x$ & put $\tan x = t$.

(x) $\int \frac{dx}{a+b \sin x}$ OR $\int \frac{dx}{a+b \cos x}$ OR $\int \frac{dx}{a+b \sin x + c \cos x}$

Hint: Convert sines & cosines into their respective tangents of half the angles, put $\tan \frac{x}{2} = t$

(xi) $\int \frac{a \cos x + b \sin x + c}{\ell \cos x + m \sin x + n} dx$. Express $Nr = A(Dr) + B \frac{d}{dx}(Dr) + c$ & proceed.

(xii) $\int \frac{x^2+1}{x^4+Kx^2+1} dx$ OR $\int \frac{x^2-1}{x^4+Kx^2+1} dx$ where K is any constant.

Hint: Divide Nr & Dr by x^2 & proceed.

(xiii) $\int \frac{dx}{(ax+b) \sqrt{px+q}}$ & $\int \frac{dx}{(ax^2+bx+c) \sqrt{px+q}}$; put $px+q = t^2$.

(xiv) $\int \frac{dx}{(ax+b) \sqrt{px^2+qx+r}}$, put $ax+b = \frac{1}{t}$; $\int \frac{dx}{(ax^2+bx+c) \sqrt{px^2+qx+r}}$, put $x = \frac{1}{t}$

(xv) $\int \sqrt{\frac{x-\alpha}{\beta-x}} dx$ or $\int \sqrt{(x-\alpha)(\beta-x)}$; put $x = \alpha \cos^2 \theta + \beta \sin^2 \theta$

$\int \sqrt{\frac{x-\alpha}{x-\beta}} dx$ or $\int \sqrt{(x-\alpha)(x-\beta)}$; put $x = \alpha \sec^2 \theta - \beta \tan^2 \theta$

$\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$; put $x-\alpha = t^2$ or $x-\beta = t^2$.